Intro Video: Section 3.7
Rates of change in the natural and social sciences

Math F251X: Calculus 1

Topic 1: position, velocity, and acceleration

Example: A particle moves according to the function $s(t) = t^4 - 4t + 1$, where position is measured in meters and time in seconds.

a) what is the velocity after 2 seconds?

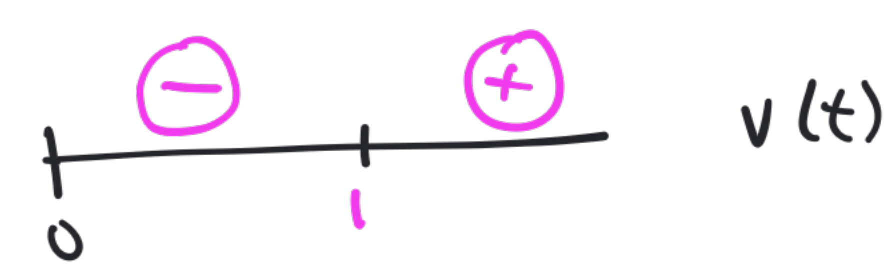
$$v(t) = s'(t) = 4t^3 - 4 = 4(t^3 - 1)$$

$$v(2) = 4(8-1) = 4(7) = 28 \text{ m/s.}$$

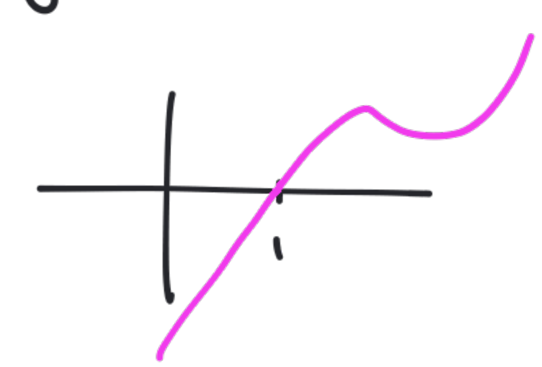
b) When is the particle at rest?

When is $v(t) = 0$: $\quad 4(t^3-1) = 0 \Rightarrow t^3 = 1 \Rightarrow \boxed{t = 1}$

c) When is the particle moving forward? $\boxed{t > 1}$



Note $v(t) = 4(t^3-1) = 4(t-1)\underbrace{(t^2+t+1)}_{\text{always positive when } t > 0}$



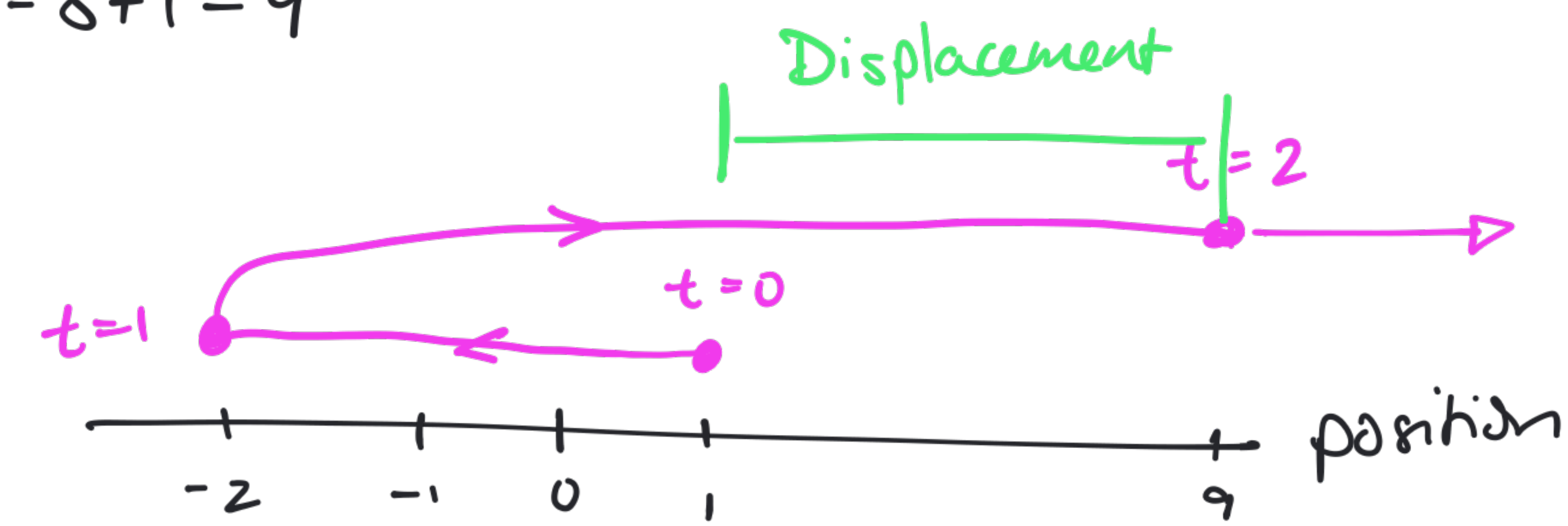
← bad graph of $v(t)$

Recall $s(t) = t^4 - 4t + 1$ and $v(t) = 4t^3 - 4 = 4(t^3 - 1)$

Draw a diagram to illustrate the motion of the particle.

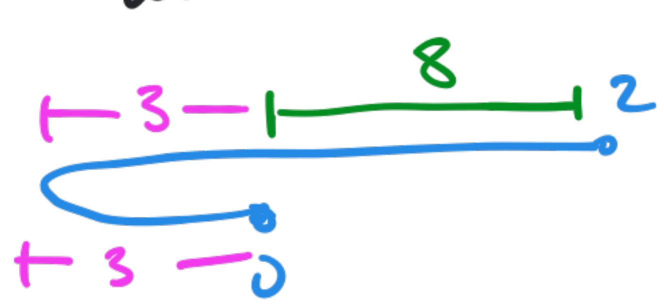
Recall changes direction at $t = 1$.

So:
$$s(0) = 1$$
$$s(1) = 1 - 4 + 1 = -2$$
$$s(2) = 16 - 4(2) + 1 = 16 - 8 + 1 = 9$$



- Displacement over the 1st 2 seconds? $9 - 1 = 8$ meters.

- Total distance traveled?



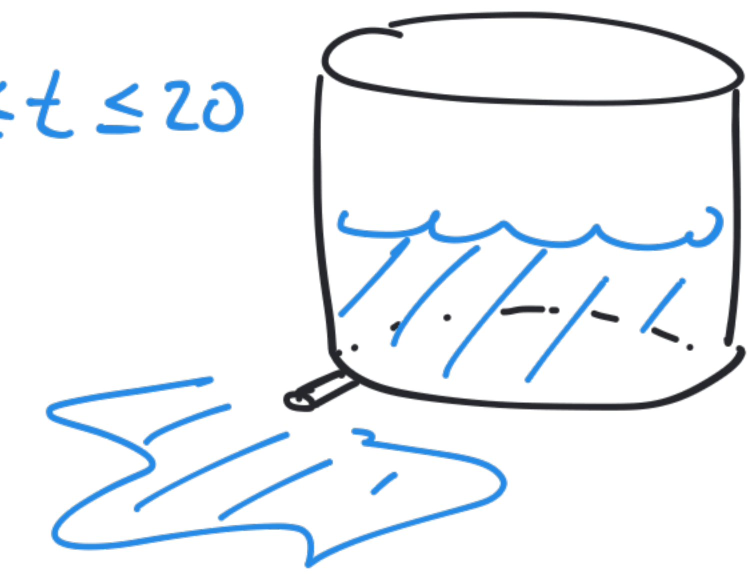
Total distance is $8 + 2(3) = 8 + 6 = 14$ meters

- Acceleration? $a(t) = v'(t) = 12t^2$ ← always positive acceleration!

Example: Suppose water drains from the bottom of a tank, and the volume is given by

$$V(t) = 1000\left(1 - \frac{1}{20}t\right)^2 . \qquad 0 \le t \le 20$$

1000 L tank



$V(0) = 1000$ ← tank full when we start!

$V(20) = 1000\left(1 - \frac{1}{20}(20)\right)^2 = 0$ ← empty after 20 minutes.

At what rate is the water flowing out after 5 minutes?

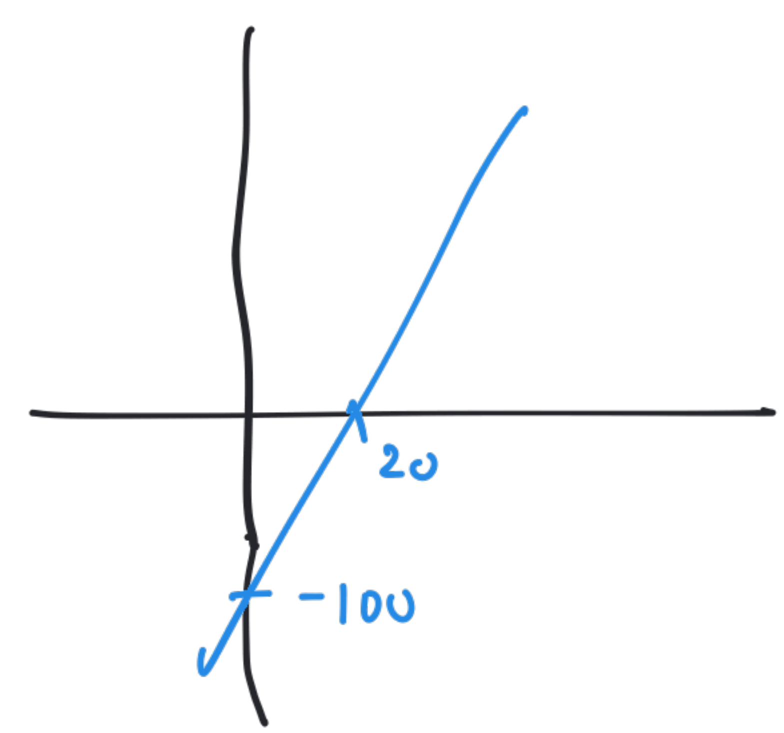
$$V'(t) = 1000(2)\left(1 - \frac{t}{20}\right)\left(-\frac{1}{20}\right) = -100\left(1 - \frac{t}{20}\right) = -100 + 5t$$

water leaving

$V'(5) = -100 + 25 = -75$ L/minute.

When is the water flowing the fastest?

Fastest at the beginning; slows down as the tank gets empty!



Marginal Cost

$C(x)$ is the total cost to make x widgets.

Marginal cost: if we go from making x_1 to x_2,

$\Delta C = C(x_2) - C(x_1)$ is the change in cost

Average rate of cost is $\dfrac{\Delta C}{\Delta x} = \dfrac{C(x_2) - C(x_1)}{x_2 - x_1}$

and as $\Delta x \to \infty$, this goes to $C'(x)$. ← marginal cost

Example: Cost to produce x yards of fabric is

$$C(x) = 1200 + 12x - 0.1x^2 + 0.0005x^3$$

What does $C'(200)$ mean? $C'(x) = 12 - 0.2x + 0.0015x^2$ so

$C'(200) = 32$. After we have made 200 yards, costs are increasing